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B.Sc. Part I (Hons) Paper - II

Differential Calculus :- Successive Differentiation

Successive differentiation is the process of differentiating a given function successively  $n$  times and the results of such differentiation are called successive derivatives. The higher order differential coefficient are of almost importance in scientific and engineering applications.

Let  $f(x)$  be a differentiable function and let its successive derivatives be denoted by  $f'(x), f''(x), \dots, f^{(n)}(x)$ .

Common notations of higher order derivatives of

$$y = f(x)$$

$$\text{1st derivative: } f'(x) \text{ or } y' \text{ or } y_1 \text{ or } \frac{dy}{dx} \text{ or } Dy$$

$$\text{2nd derivative: } f''(x) \text{ or } y'' \text{ or } y_2 \text{ or } \frac{d^2y}{dx^2} \text{ or } D^2y$$

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$$\text{nth derivative: } f^{(n)}(x) \text{ or } y^{(n)} \text{ or } y_n \text{ or } \frac{d^ny}{dx^n} \text{ or } D^ny$$

Ex ① Find  $n$ th derivative of  $e^{ax}$

Soln. let  $y = e^{ax}$

$$y_1 = ae^{ax}$$

$$y_2 = a^2e^{ax}$$

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$$y_n = a^n e^{ax}$$

② Find  $n$ th derivative of  $(ax+b)^m$ ,  $m$  is a true integer greater than  $n$ .

Soln. Let  $y = (ax+b)^m$

$$y_1 = ma(ax+b)^{m-1}$$

$$y_2 = m(m-1)a^2(ax+b)^{m-2}$$

⋮

$$y_n = m(m-1)\dots(m-n+1)a^n(ax+b)^{m-n}$$

$$= \frac{m!}{(m-n)!} a^n (ax+b)^{m-n}$$

③ Find  $n$ th derivative of  $y = \log(ax+b)$

Soln. Let  $y = \log(ax+b)$

$$y_1 = \frac{a}{ax+b}$$

$$y_2 = \frac{-a^2}{(ax+b)^2}$$

$$y_3 = \frac{2a^3}{(ax+b)^3}$$

⋮

$$y_n = (-1)^{n-1} \frac{(n-1)! a^n}{(ax+b)^n}$$

④ Find  $n$ th derivative of  $y = \sin(ax+b)$

Soln. Let  $y = \sin(ax+b)$

$$y_1 = a \cos(ax+b) = a \sin(ax+b + \frac{\pi}{2})$$

$$y_2 = a^2 \cos(ax+b + \frac{\pi}{2}) = a^2 \sin(ax+b + \frac{2\pi}{2})$$

⋮

$$y_n = a^n \sin(ax+b + \frac{n\pi}{2})$$

Similarly  $y = \cos(ax+b)$  the  $y_n = a^n \cos(ax+b + \frac{n\pi}{2})$ .